

CONTRIBUTIONS FROM THE OBSERVATORY OF  
COLUMBIA COLLEGE, NEW YORK.—No. 6.

JOHN K. REES, Director.

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THE  
PARALLAX OF  $\eta$  CASSIOPEIÆ,

DEDUCED FROM THE

RUTHERFURD PHOTOGRAPHIC MEASURES.

BY  
HERMAN S. DAVIS.

Submitted in partial fulfillment of the requirements for the degree of Doctor  
of Philosophy in the University Faculty of Pure Science,  
Columbia College.

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NEW YORK:  
1895.

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## PREFACE.

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On November 13, 1890, Lewis M. Rutherfurd, LL. D., of New York City, presented to the Columbia College Observatory his complete set of original negatives of the Sun, the Moon, and many star clusters; also some thirty quarto volumes of measures of certain star plates. A catalogue of the plates has been published in the ANNALS OF THE NEW YORK ACADEMY OF SCIENCES, VI., June, 1891, p. 1.

At the same time Mr. Rutherfurd authorized me to arrange for the reduction of the measures and the publication of the same.

After Mr. Rutherfurd's death, on May 30, 1892, his son, Rutherfurd Stuyvesant generously supplied the means for continuing the work.

This Observatory has published up to the present time in the *Contributions from the Observatory of Columbia College, New York*, No. 3, The Rutherfurd Photographic Measures of the Group of the Pleiades, by Harold Jacoby; No. 4, The Rutherfurd Photographic Measures of the Stars about  $\beta$  Cygni, by Harold Jacoby; No. 5, The Parallaxes of  $\mu$  and  $\theta$  Cassiopeiæ, deduced from the Rutherfurd Photographic Measures, by Harold Jacoby; and now issues No. 6, on The Parallax of  $\eta$  Cassiopeiæ, by Herman S. Davis.

JOHN K. REES,

*Professor of Astronomy  
and Director of the Observatory,  
Columbia College.*

NEW YORK CITY, February 28, 1895.



# XI.—*The Parallax of $\eta$ Cassiopeiæ, deduced from Rutherford Photographic Measures.*

BY HERMAN S. DAVIS.

Read Feb. 4, 1895.

Between July 30, 1870, and December 21, 1873, twenty-seven negatives of the stars about  $\eta$  Cassiopeiæ were taken by RUTHERFURD. The conditions of exposure and, indeed, even the dates, as well as also the methods of measurement and of reduction are almost identical with those of  $\mu$  and  $\theta$  Cassiopeiæ already reduced for parallax.\* Hence reference to that paper will make clear the plan followed in the application of corrections for division errors, the "tangent correction," for refraction and for aberration.

The observations have been reduced to the epoch 1872.0, using AUWERS' proper motion which is

$$\Delta\alpha = + 0^s.1346 \qquad \Delta\delta = - 0.''481,$$

corresponding to a motion of  $1.''1965$  on a great circle whose position angle is  $113^\circ 42' 10''$ . Representing this motion by  $\rho$  (expressed in terms of one division of the glass scale as a unit of measure =  $0^d.042712$ ) and the angle by  $\chi$  at the time 1872.0, the time of observation by  $t$ , the position angle of the star relative to  $\eta$  Cassiopeiæ by  $p$  and its distance by  $s$  after the corrections named in the first paragraph have been applied, we compute†

$$\tau = t - 1872.$$

$$S_1 = \cos (\chi - p)$$

$$S_2 = - \frac{\tau}{2s} \sin^2 (\chi - p)$$

$$P_1 = \tau\rho$$

$$P_2 = \tau^2\rho^2$$

\*The Parallaxes of  $\mu$  and  $\theta$  Cassiopeiæ, deduced from Rutherford Photographic Measures. By Harold Jacoby. *Annals N. Y. Acad. Sci.*, Vol. VIII., p. 1.

†*Ibid.*, p. 3.

The numerical values of these quantities as applicable to the present purpose are given in Table III., pages 308-9, and the correction for proper motion is

$$S_1 P_1 + S_2 P_2$$

to be added to the observed distance  $s$ .

The distances thus completely corrected are to be found in the second and third columns of Table IV., pages 310-15. The fourth and fifth columns respectively of the same table contain the *sum* of the distances of the two comparison stars from  $\gamma$  Cassiopeiæ and the *difference* of that sum from the mean given at the foot of the fourth column. In the sixth, seventh, and eighth columns are placed the *difference* of the distances as given in the second and third columns; the *scale correction*, which is simply a proportional part of the quantity given in the column "mean *minus* sum;" and finally the *corrected difference* to be used in forming the parallax equations. The latter equations, together with their solutions, are in Table V., pages 316-21, where the absolute terms are expressed in units of the second decimal place equivalent to 0."2801 as the value of one division of the glass scale.

Using AUWERS' values of the coördinates of  $\gamma$  Cassiopeiæ reduced to 1872.0

$$\alpha = 0^h 41^m 22.^s 108$$

$$\delta = 57^\circ 8' 10.'' 50$$

and the almanac values of  $r$  and  $\odot$ , the radius vector and longitude of the sun respectively, the values of  $S_3$ ,  $S_4$ ,  $P_3$  and  $P_4$  were computed by the formulæ\*

$$\begin{aligned} g \sin G &= \sin \delta \cos \alpha & h \sin H &= \sin \delta \sin \alpha \\ g \cos G &= \sin \alpha & h \cos H &= -\cos \delta \end{aligned}$$

$$\begin{aligned} f \sin F &= h \sin (H + \varepsilon) \\ f \cos F &= -\cos \alpha \cos \varepsilon \end{aligned}$$

$$S_3 = f \sin (p + F)$$

$$S_4 = g \sin (p + G)$$

$$P_3 = -r \sin \odot$$

$$P_4 = -r \cos \odot$$

\*Annals N. Y. Acad. Sci., Vol. VIII., p. 4.



The resulting values for the separate stars and plates are given in Table III., pages 308-9. From them we derive as the parallax coefficient for any one of the equations of Table V.

$$(S_3 - S'_3) P_3 + (S_4 - S'_4) P_4$$

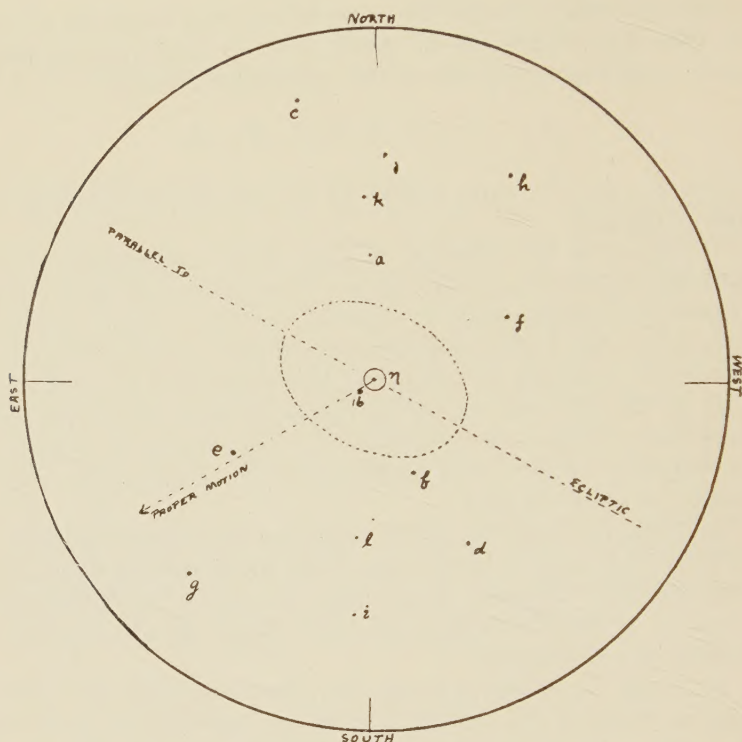
where  $S'_3$  and  $S'_4$  refer always to the *less distant* comparison star of the pair.

It seemed to me desirable, in view of the exceedingly great difficulty of getting comparison stars suitably situated either with respect to position angle or distance, to take a larger number than usual, and I have consequently reduced six pairs, being *all* that were impressed on the plates sufficiently often in both seasons of the year, and the present paper includes all six, though only three of them are so situated with reference to the parallactic ellipse as to give good coefficients for  $\pi$ . Every observation and every plate was used.

In Table II., page 308, will be found the approximate positions of these stars relative to the central star which the accompanying diagram also represents graphically, and shows in addition the direction of the axes of the parallactic ellipse and of the proper motion of  $\eta$  Cassiopeiae.

It should be stated, however, that the circle does not represent the limits of the field of view, nor are the distances given on the same scale as those of the plates at the focus of the telescope; the diagram is intended to give only roughly the *relative* location of the stars used in this investigation. It will be noticed how the N. E. quadrant is inconveniently void of stars bright enough to be impressed on the plates at all seasons of the year.

The star designated as No. 16 was so numbered by RUTHERFURD on the plates when measured, but is the same star as No. 45, in order of right ascension, referred to on page 305.



The results are :

| Comp. Stars.          | $\pi$              | Weight. | $y$                | Prob. error—<br>one equation. |
|-----------------------|--------------------|---------|--------------------|-------------------------------|
| <i>a</i> and <i>b</i> | $+0.349 \pm 0.086$ | 21.7800 | $+0.260 \pm 0.081$ | $\pm 0.403$                   |
| <i>c</i> and <i>d</i> | $+0.385 \pm 0.084$ | 28.9724 | $+0.165 \pm 0.085$ | $\pm 0.362$                   |
| <i>e</i> and <i>f</i> | $+0.568 \pm 0.056$ | 29.6264 | $+0.378 \pm 0.078$ | $\pm 0.305$                   |
| <i>g</i> and <i>h</i> | $+0.662 \pm 0.078$ | 8.2443  | $+0.232 \pm 0.049$ | $\pm 0.2740$                  |
| <i>i</i> and <i>j</i> | $+0.660 \pm 0.138$ | 6.1457  | $-0.111 \pm 0.066$ | $\pm 0.342$                   |
| <i>k</i> and <i>l</i> | $+0.297 \pm 0.155$ | 6.1365  | $-0.070 \pm 0.073$ | $\pm 0.384$                   |

From these we get as the value of the parallax of  $\eta$  Cassiopeiæ

$$\pi = +0.443 \pm 0.043 \quad \text{using the first three pairs only}$$

$$\text{or} \quad = +0.465 \pm 0.044 \quad \text{using all six pairs.}$$

It would be quite unsafe to conclude that the small result derived from the stars *k* and *l* is due to a parallax of one or other

of these two; yet it may not be amiss to state that if we combine, to form a comparison pair,

|                     |                              |               |                    |                |
|---------------------|------------------------------|---------------|--------------------|----------------|
| $l$ with $a$ we get | $\pi = +0''.282 \pm 0''.197$ | weight 5.1956 | p. e. one equation | $\pm 0''.4492$ |
| $i$ with $k$        | $\pi = +0.359 \pm 0.124$     | 8.0319        |                    | $\pm 0.3516$   |

Results by other observers for the parallax of  $\eta$  Cassiopeiae are :

|            |                              |                               |
|------------|------------------------------|-------------------------------|
| O. Struve* | $\pi = +0''.373 \pm 0''.098$ | by measures of position angle |
|            | $= +0.096 \pm 0.051$         | " " " distance                |
|            | concluded mean               | $+0''.154 \pm 0''.045$        |

|                         |                                |                               |
|-------------------------|--------------------------------|-------------------------------|
| Schweizer† and Socoloff | $\pi = +0''.1386 \pm 0''.0849$ | by measures of position angle |
|                         | $= +0.3743 \pm 0.0723$         | " " " distance                |
|                         | concluded mean                 | $+0''.2750 \pm 0''.0551$      |

STRUVE is not explicit in his statement, but the evidence seems to be that his comparison star was the same as that used by SCHWEIZER, namely, No. 45 on these plates, and whose position, as given by the RUTHERFURD measures,‡ is

$$\begin{aligned} \alpha &= 0^h 41^m 49^s.555 \\ \delta &= 57^\circ 6' 8.''87 \end{aligned}$$

reduced from twelve plates whose mean epoch is 1873.016 referred to  $\eta$  Cassiopeiae at the epoch 1872.0 assuming its coördinates at that date to be as given on page 302. This is DM.  $57^\circ 155$ .

From the values of  $y$  obtained in the solution of the normal equations a correction for the assumed proper motion of  $\eta$  Cassiopeiae may be computed from equations of the form§

$$(\cos p - \cos p') w + (\sin p - \sin p') v - y = 0$$

where  $w$  = the correction required by the adopted value of  $\rho \cos \chi$   
 $v$  = " " " " " "  $\rho \sin \chi$

and  $p, p'$  = the position angle of the comparison stars, the primed letter referring always to the *less distant* of the two.

\* Bulletin Physico-Mathématique de l'Académie de Saint-Petersbourg, vol. xiv., p. 228, published 1855.

† Annales de l'Observatoire de Moscou, vol. viii., part 2, p. 89, published 1882.

‡ The Rutherford Photographic Measures of Sixty-two Stars about  $\eta$  Cassiopeiae. By Herman S. Davis. Contrib. from the Observ. of Columbia College, No. 7.

§ Annals N. Y. Acad. Sci.; Vol. VIII., p. 10.

The equations thus obtained are :

$$\begin{array}{rclcl}
 \text{Stars } a, b & + 1.9146 w & + 0.4593 v & - 0.2598 & = 0 \\
 c, d & + 1.8232 & + 0.7773 & - 0.1653 & = 0 \\
 e, f & - 0.8337 & + 0.8176 & - 0.3781 & = 0 \\
 g, h & - 1.5436 & + 1.2615 & - 0.2318 & = 0 \\
 i, j & - 1.9961 & + 0.1171 & + 0.1113 & = 0 \\
 k, l & + 1.9950 & - 0.0461 & + 0.0698 & = 0
 \end{array}$$

from which the normal equations are :

$$\begin{array}{rcl}
 + 18.0319 w & - 1.4917 v & - 0.2087 = 0 \\
 & + 5.7260 & - 1.2176 = 0
 \end{array}$$

and the solution gives :

$$\begin{array}{l}
 w = + 0.''0298 \pm 0.''0147 \\
 v = + 0.2204 \pm 0.0264
 \end{array}$$

the probable error of one equation  $= \pm 0.''0624$

Applying these corrections to the values previously assumed,

$$\text{Corrected } \rho \sin \chi = + 1.''3160 = \Delta a \cos \delta$$

$$\text{" } \rho \cos \chi = - 0.4512 = \Delta \delta$$

$$\text{which gives } \Delta a = + 0^s.1617 \quad \Delta \delta = - 0.''4512$$

corresponding to a motion of  $1.''3912$  on a great circle whose position angle is  $108^\circ 55' 30''$ .

Considering the difficulties that beset the investigation of the parallax of this star, and the probability that all previous determinations have been made with a single comparison star, which may itself have a parallax; considering also the large differences exhibited by the separate results of other observers, it would seem that the result here given is not impossibly large, if the RUTHERFURD plates are subject to no systematic error. Whether such error exist or not, these plates of  $\eta$  Cassiopeiæ seem at any rate entirely unsuited for an investigation of it, and I have therefore in the preparation of this paper confined myself strictly to the methods and formulæ and often even to the phraseology of Professor Jacoby in his paper on  $\mu$  and  $\theta$  Cassiopeiæ, already several times referred to, since this is but one in a series of many papers to be produced from the Rutherford measures, and since in such a series uniformity is desirable.

A catalogue of all the stars (62) on these plates has been reserved for a separate paper which will be published soon, as No. 7 of *Contributions from the Observatory of Columbia College*.



TABLE I.—GENERAL DATA.

OBSERVATORY OF L. M. RUTHERFURD, NEW YORK.

Lat. =  $40^{\circ} 43' 48''.5$ . Long. =  $4^{\text{h}} 55^{\text{m}} 56^{\text{s}}$ . 62 W.

| Plate No. | Date.        | Sidereal Time.                                  | Hour Angle.                                     | Zen. Dist. | Parall. Angle. | Ext. Temp. | Focal Mic'r. |
|-----------|--------------|-------------------------------------------------|-------------------------------------------------|------------|----------------|------------|--------------|
| 1         | 1870 July 30 | 21 <sup>h</sup> 32 <sup>m</sup> 15 <sup>s</sup> | 20 <sup>h</sup> 50 <sup>m</sup> 53 <sup>s</sup> | 34.20      | — 97.95        | 70         | 7.9          |
| 2         | 1870 July 30 | 22 7 30                                         | 21 26 8                                         | 29.55      | — 107.02       | 70         | 7.9          |
| 3         | 1870 Aug. 4  | 23 53 55                                        | 23 12 33                                        | 18.11      | — 149.92       | 70         | 7.9          |
| 4         | 1870 Aug. 4  | 23 7 50                                         | 22 26 28                                        | 22.26      | — 127.44       | 70         | 7.9          |
| 5         | 1871 July 17 | 20 2 20                                         | 19 20 58                                        | 46.34      | — 79.34        | 72         | 8.0          |
| 6         | 1871 July 17 | 20 54 35                                        | 20 13 13                                        | 39.30      | — 89.58        | 72         | 8.0          |
| 7         | 1871 Dec. 15 | 4 44 45                                         | 4 3 23                                          | 41.55      | + 86.20        | 23         | 8.0          |
| 8         | 1872 Jan. 2  | 1 44 15                                         | 1 2 53                                          | 19.71      | + 141.56       | 34         | 7.9          |
| 9         | 1872 Jan. 2  | 2 23 40                                         | 1 42 18                                         | 23.23      | + 123.93       | 34         | 7.9          |
| 10        | 1872 Jan. 5  | 3 34 35                                         | 2 53 13                                         | 32.10      | + 101.87       | 35         | 7.9          |
| 11        | 1872 June 30 | 20 42 38                                        | 20 1 16                                         | 40.90      | — 87.14        | 78         | 7.1          |
| 12        | 1872 June 30 | 21 13 28                                        | 20 32 6                                         | 36.74      | — 93.65        | 78         | 7.1          |
| 13        | 1872 July 19 | 19 32 52                                        | 18 51 30                                        | 50.24      | — 73.98        | 71         | 7.29         |
| 14        | 1872 July 19 | 20 9 28                                         | 19 28 6                                         | 45.38      | — 80.67        | 71         | 7.29         |
| 15        | 1873 Jan. 9  | 3 38 48                                         | 2 57 26                                         | 32.64      | + 100.81       | 24         | 7.8          |
| 16        | 1873 Jan. 10 | 2 18 2                                          | 1 36 40                                         | 22.60      | + 126.17       | 21         | 7.95         |
| 17        | 1873 Jan. 10 | 2 49 58                                         | 2 8 36                                          | 26.34      | + 114.64       | 21         | 7.95         |
| 18        | 1873 July 15 | 19 51 58                                        | 19 10 36                                        | 47.72      | — 77.43        | 75         | 7.75         |
| 19        | 1873 July 15 | 20 21 18                                        | 19 39 56                                        | 43.79      | — 82.93        | 75         | 7.75         |
| 20        | 1873 July 20 | 19 23 18                                        | 18 41 56                                        | 51.48      | — 72.27        | 69         | 7.8          |
| 21        | 1873 July 20 | 19 59 32                                        | 19 18 10                                        | 46.71      | — 78.82        | 69         | 7.8          |
| 22        | 1873 July 21 | 19 31 32                                        | 18 50 10                                        | 50.42      | — 73.75        | 69         | 7.8          |
| 23        | 1873 Dec. 15 | 1 43 8                                          | 1 1 46                                          | 19.20      | + 142.14       | 40         | 7.8          |
| 24        | 1873 Dec. 15 | 2 23 32                                         | 1 42 10                                         | 23.21      | + 123.99       | 40         | 7.8          |
| 25        | 1873 Dec. 18 | 2 54 2                                          | 2 12 40                                         | 26.85      | + 113.34       | 41         | 7.8          |
| 26        | 1873 Dec. 18 | 3 38 48                                         | 2 57 26                                         | 32.64      | + 100.81       | 41         | 7.8          |
| 27        | 1873 Dec. 21 | 2 4 32                                          | 1 23 10                                         | 21.18      | + 131.87       | 27         | 7.9          |

TABLE II.—COMPARISON STARS.

| Designation<br>of Comp.<br>Star. | No. in<br>A. G. Cat.<br>Helsingfors-<br>Gotha. | Mag. in<br>A. G. Cat. | Approx. Position Referred to $\gamma$ Cass. |                 |
|----------------------------------|------------------------------------------------|-----------------------|---------------------------------------------|-----------------|
|                                  |                                                |                       | Distance.                                   | Position Angle. |
| <i>a</i>                         | 658                                            | 8.9                   | 1316''                                      | $3^{\circ} 23'$ |
| <i>b</i>                         | 639                                            | 8.8                   | 1148                                        | 203 36          |
| <i>c</i>                         | 687                                            | 8.2                   | 3416                                        | 15 24           |
| <i>d</i>                         | 621                                            | 8.8                   | 2100                                        | 210 47          |
| <i>e</i>                         | 713                                            | 9.0                   | 1820                                        | 113 34          |
| <i>f</i>                         | 609                                            | 8.4                   | 1708                                        | 295 43          |
| <i>g</i>                         | 730                                            | 7.5                   | 3052                                        | 136 10          |
| <i>h</i>                         | 607                                            | 9.0                   | 2828                                        | 325 19          |
| <i>i</i>                         | 665                                            | 7.1                   | 2744                                        | 175 18          |
| <i>j</i>                         | 649                                            | 8.8                   | 2604                                        | 357 59          |
| <i>k</i>                         | 659                                            | 8.8                   | 2156                                        | 2 53            |
| <i>l</i>                         | 663                                            | 8.3                   | 1848                                        | 174 28          |

TABLE III.—PROPER MOTION AND PARALLAX.

| Star.    | Proper Motion. |         | Parallax. |         |
|----------|----------------|---------|-----------|---------|
|          | $S_1$ .        | $S_2$ . | $S_3$ .   | $S_4$ . |
| <i>a</i> | —0.347         | —0.009  | —0.1307   | 0.8356  |
| <i>b</i> | 0.002          | —0.012  | 0.4324    | —0.8290 |
| <i>c</i> | —0.144         | —0.004  | —0.3145   | 0.8443  |
| <i>d</i> | —0.123         | —0.007  | 0.5285    | —0.8019 |
| <i>e</i> | —1.000         | 0.000   | —0.7962   | —0.1660 |
| <i>f</i> | —0.999         | 0.000   | 0.7793    | —0.1969 |
| <i>g</i> | —0.924         | —0.001  | —0.5690   | —0.4719 |
| <i>h</i> | —0.852         | —0.001  | 0.4496    | 0.5775  |
| <i>i</i> | 0.476          | —0.004  | —0.0034   | —0.8089 |
| <i>j</i> | —0.434         | —0.004  | —0.0458   | —0.8196 |
| <i>k</i> | —0.355         | —0.006  | —0.1229   | —0.8345 |
| <i>l</i> | —0.488         | —0.006  | —0.0097   | —0.8052 |

TABLE III (*continued*).—PROPER MOTION AND PARALLAX.

| Plate<br>No. | $t-1872.$   | Proper Motion. |              | Parallax.    |              |
|--------------|-------------|----------------|--------------|--------------|--------------|
|              |             | $P_1.$         | $P_2.$       | $P_3.$       | $P_4.$       |
| 1            | -1.420      | -0.0606        | $\mp$ 0.0037 | -0.8024      | $\mp$ 0.6213 |
| 2            | -1.420      | -0.0606        | $\mp$ 0.0037 | -0.8024      | $\mp$ 0.6213 |
| 3            | -1.406      | -0.0600        | $\mp$ 0.0036 | -0.7485      | $\mp$ 0.6843 |
| 4            | -1.406      | -0.0600        | $\mp$ 0.0036 | -0.7485      | $\mp$ 0.6843 |
| 5            | -0.455      | -0.0195        | $\mp$ 0.0004 | -0.9210      | $\mp$ 0.4295 |
| 6            | -0.455      | -0.0195        | $\mp$ 0.0004 | -0.9210      | $\mp$ 0.4295 |
| 7            | -0.043      | -0.0018        | 0.0000       | -0.9783      | -0.1054      |
| 8            | 0.006       | $\mp$ 0.0002   | 0.0000       | $\mp$ 0.9610 | -0.2078      |
| 9            | 0.006       | $\mp$ 0.0002   | 0.0000       | $\mp$ 0.9610 | -0.2078      |
| 10           | 0.014       | -0.0006        | 0.0000       | $\mp$ 0.9487 | -0.2586      |
| 11           | -0.501      | $\mp$ 0.0214   | $\mp$ 0.0005 | -1.0025      | $\mp$ 0.1696 |
| 12           | -0.501      | $\mp$ 0.0214   | $\mp$ 0.0005 | -1.0025      | $\mp$ 0.1696 |
| 13           | -0.553      | $\mp$ 0.0236   | $\mp$ 0.0006 | -0.8994      | $\mp$ 0.4725 |
| 14           | -0.553      | $\mp$ 0.0236   | $\mp$ 0.0006 | -0.8994      | $\mp$ 0.4725 |
| 15           | -1.025      | $\mp$ 0.0438   | $\mp$ 0.0019 | 0.9235       | -0.3380      |
| 16           | -1.028      | $\mp$ 0.0439   | $\mp$ 0.0019 | $\mp$ 0.9173 | -0.3546      |
| 17           | -1.028      | $\mp$ 0.0439   | $\mp$ 0.0019 | -0.9173      | -0.3546      |
| 18           | $\mp$ 1.539 | $\mp$ 0.0657   | $\mp$ 0.0043 | -0.9308      | $\mp$ 0.4080 |
| 19           | $\mp$ 1.539 | $\mp$ 0.0657   | $\mp$ 0.0043 | -0.9308      | $\mp$ 0.4080 |
| 20           | 1.553       | $\mp$ 0.0663   | $\mp$ 0.0044 | -0.8920      | $\mp$ 0.4863 |
| 21           | 1.553       | $\mp$ 0.0663   | $\mp$ 0.0044 | -0.8920      | $\mp$ 0.4863 |
| 22           | 1.556       | $\mp$ 0.0664   | $\mp$ 0.0044 | -0.8850      | $\mp$ 0.4987 |
| 23           | $\mp$ 1.956 | $\mp$ 0.0836   | $\mp$ 0.0070 | $\mp$ 0.9792 | $\mp$ 0.0966 |
| 24           | -1.956      | $\mp$ 0.0836   | $\mp$ 0.0070 | -0.9792      | $\mp$ 0.0966 |
| 25           | $\mp$ 1.965 | $\mp$ 0.0839   | $\mp$ 0.0070 | $\mp$ 0.9827 | $\mp$ 0.0441 |
| 26           | $\mp$ 1.965 | $\mp$ 0.0839   | $\mp$ 0.0070 | -0.9827      | $\mp$ 0.0441 |
| 27           | -1.973      | $\mp$ 0.0843   | $\mp$ 0.0071 | -0.9835      | -0.0083      |

TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS *a* AND *b*.

| Plate<br>No. | Corrected Distance. |                 | Sum<br><i>a</i> + <i>b</i> . | Mean<br>Minus Sum | Difference<br><i>a</i> — <i>b</i> . | Scale Corr. | Corrected<br>Difference. |
|--------------|---------------------|-----------------|------------------------------|-------------------|-------------------------------------|-------------|--------------------------|
|              | Star <i>a</i> .     | Star <i>b</i> . |                              |                   |                                     |             |                          |
| 1            | 47.2431             | 40.7833         | 88.0264                      | — .0084           | 6.4598                              | — .0017     | 6.4581                   |
| 4            | .2506               | .7693           | 88.0199                      | — .0019           | .4813                               | — .0004     | .4809                    |
| 5            | .2522               | .7816           | 88.0338                      | — .0158           | .4706                               | — .0032     | .4674                    |
| 6            | .2382               | .7841           | 88.0223                      | — .0043           | .4541                               | — .0009     | .4532                    |
| 7            | .2620               | .7327           | 87.9947                      | + .0233           | .5293                               | + .0047     | .5340                    |
| 8            | .2514               | .7583           | 88.0097                      | + .0083           | .4931                               | + .0017     | .4948                    |
| 10           | .2541               | .7492           | 88.0033                      | + .0147           | .5049                               | + .0029     | .5078                    |
| 11           | .2236               | .7742           | 87.9978                      | + .0202           | .4494                               | + .0040     | .4534                    |
| 12           | .2426               | .7731           | 88.0157                      | + .0023           | .4695                               | + .0005     | .4700                    |
| 13           | .2626               | .7780           | 88.0406                      | — .0226           | .4846                               | — .0045     | .4801                    |
| 14           | .2714               | .7650           | 88.0364                      | — .0184           | .5064                               | — .0037     | .5027                    |
| 15           | .2570               | .7412           | 87.9982                      | — .0198           | .5158                               | + .0040     | .5198                    |
| 16           | .2544               | .7470           | 88.0014                      | + .0166           | .5074                               | + .0033     | .5107                    |
| 17           | .2495               | .7483           | 87.9978                      | + .0202           | .5012                               | + .0040     | .5052                    |
| 18           | .2588               | .7796           | 88.0384                      | — .0204           | .4792                               | — .0041     | .4751                    |
| 19           | .2605               | .7734           | 88.0339                      | — .0159           | .4871                               | — .0032     | .4839                    |
| 20           | .2497               | .7824           | 88.0321                      | — .0141           | .4673                               | — .0028     | .4645                    |
| 21           | .2491               | .7776           | 88.0267                      | — .0087           | .4715                               | — .0017     | .4698                    |
| 22           | .2500               | .7828           | 88.0328                      | — .0148           | .4672                               | — .0030     | .4642                    |
| 23           | .2371               | .7802           | 88.0173                      | + .0007           | .4569                               | — .0001     | .4570                    |
| 24           | .2289               | .7862           | 88.0151                      | + .0029           | .4427                               | + .0006     | .4433                    |
| 25           | .2356               | .7737           | 88.0093                      | + .0087           | .4619                               | + .0017     | .4636                    |
| 26           | .2324               | .7818           | 88.0142                      | + .0038           | .4506                               | + .0008     | .4514                    |
| 27           | .2366               | .7778           | 88.0144                      | + .0036           | .4588                               | + .0007     | .4595                    |
| Mean,        |                     |                 | 88.0180                      |                   | Assumed Value, 6.4800               |             |                          |



TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS *c* AND *d*.

| Plate No. | Corrected Distance. |                | Sum.<br><i>c + d.</i> | Mean<br><i>Minus</i> Sum | Difference<br><i>c - d.</i> | Scale Corr. | Corrected Difference. |
|-----------|---------------------|----------------|-----------------------|--------------------------|-----------------------------|-------------|-----------------------|
|           | Star <i>c.</i>      | Star <i>d.</i> |                       |                          |                             |             |                       |
| 4         | 121.4926            | 75.2954        | 196.7880              | — .0202                  | 46.1972                     | — .0040     | 46.1932               |
| 6         | .4826               | .3091          | .7917                 | — .0239                  | .1735                       | — .0048     | .1687                 |
| 7         | .5033               | .2631          | .7664                 | + .0014                  | .2402                       | + .0003     | .2405                 |
| 8         | .4960               | .2684          | .7644                 | + .0034                  | .2276                       | + .0007     | .2283                 |
| 9         | .4978               | .2661          | .7639                 | + .0039                  | .2317                       | + .0008     | .2325                 |
| 10        | .4954               | .2605          | .7559                 | + .0119                  | .2349                       | + .0024     | .2373                 |
| 11        | .4711               | .2842          | .7553                 | + .0125                  | .1869                       | + .0025     | .1894                 |
| 12        | .4750               | .2853          | .7603                 | + .0075                  | .1897                       | + .0015     | .1912                 |
| 15        | .4903               | .2538          | .7441                 | + .0237                  | .2365                       | + .0047     | .2412                 |
| 16        | .4996               | .2540          | .7536                 | + .0142                  | .2456                       | + .0028     | .2484                 |
| 17        | .4969               | .2505          | .7474                 | + .0204                  | .2464                       | + .0041     | .2505                 |
| 18        | .4996               | .2854          | .7850                 | — .0172                  | .2142                       | — .0034     | .2108                 |
| 19        | .5000               | .2979          | .7979                 | — .0301                  | .2021                       | — .0060     | .1961                 |
| 20        | .5016               | .2807          | .7823                 | — .0145                  | .2209                       | — .0029     | .2180                 |
| 22        | .4832               | .2985          | .7817                 | — .0139                  | .1847                       | — .0028     | .1819                 |
| 23        | .4906               | .2759          | .7665                 | + .0013                  | .2147                       | + .0003     | .2150                 |
| 24        | .4764               | .2933          | .7697                 | — .0019                  | .1831                       | — .0004     | .1827                 |
| 25        | .4778               | .2894          | .7672                 | + .0006                  | .1884                       | + .0001     | .1885                 |
| 26        | .4766               | .2762          | .7528                 | + .0150                  | .2004                       | + .0030     | .2034                 |
| 27        | .4779               | .2836          | .7615                 | + .0063                  | .1943                       | + .0013     | .1956                 |
|           |                     | Mean, 196.7678 |                       |                          | Assumed Value,              |             | 46.2100               |

TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS *e* AND *f*.

| Plate<br>No.   | Corrected Distance. |                 | Sum<br><i>e</i> + <i>f</i> . | Mean<br><i>minus</i> Sum | Difference<br><i>e</i> - <i>f</i> . | Scale Corr. | Corrected<br>Difference. |
|----------------|---------------------|-----------------|------------------------------|--------------------------|-------------------------------------|-------------|--------------------------|
|                | Star <i>e</i> .     | Star <i>f</i> . |                              |                          |                                     |             |                          |
| 1              | 64.7454             | 60.4758         | 125.2212                     | — .0167                  | 4.2696                              | — .0017     | 4.2679                   |
| 6              | .7406               | .4776           | 2182                         | — .0137                  | .2630                               | — .0013     | .2617                    |
| 8              | .7483               | .4524           | .2007                        | + .0038                  | .2959                               | + .0004     | .2963                    |
| 9              | .7563               | .4531           | .2094                        | — .0049                  | .3032                               | — .0005     | .3027                    |
| 11             | .7099               | .4854           | .1953                        | + .0092                  | .2245                               | + .0009     | .2254                    |
| 12             | .7082               | .4911           | .1993                        | + .0052                  | .2171                               | + .0005     | .2176                    |
| 13             | .7010               | .5089           | .2099                        | — .0054                  | .1921                               | — .0005     | .1916                    |
| 14             | .7174               | .4965           | .2139                        | — .0094                  | .2209                               | — .0009     | .2200                    |
| 15             | .7367               | .4512           | .1879                        | + .0166                  | .2855                               | + .0017     | .2872                    |
| 16             | .7250               | .4599           | .1849                        | + .0196                  | .2651                               | + .0020     | .2671                    |
| 17             | .7221               | .4584           | .1805                        | + .0240                  | .2637                               | + .0024     | .2661                    |
| 18             | .7092               | .4969           | .2061                        | — .0016                  | .2123                               | — .0002     | .2121                    |
| 20             | .7169               | .4809           | .1978                        | + .0067                  | .2360                               | + .0007     | .2367                    |
| 22             | .7300               | .4811           | .2111                        | — .0066                  | .2489                               | — .0007     | .2482                    |
| 23             | .7449               | .4756           | .2205                        | — .0160                  | .2693                               | — .0016     | .2677                    |
| 24             | .7456               | .4676           | .2132                        | — .0087                  | .2780                               | — .0009     | .2771                    |
| 25             | .7322               | .4738           | .2060                        | — .0015                  | .2584                               | — .0002     | .2582                    |
| 27             | .7396               | .4656           | .2052                        | — .0007                  | .2740                               | — .0001     | .2739                    |
| Mean, 125.2045 |                     |                 |                              |                          | Assumed Value, 4.2540               |             |                          |

TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS  $g$  AND  $h$ .

| Plate<br>No.   | Corrected Distance. |            | Sum<br>$g + h$ .      | Mean<br><i>minus</i> Sum | Difference<br>$g - h$ . | Scale Corr. | Corrected<br>Difference. |
|----------------|---------------------|------------|-----------------------|--------------------------|-------------------------|-------------|--------------------------|
|                | Star $g$ .          | Star $h$ . |                       |                          |                         |             |                          |
| 1              | 108.3264            | 100.6776   | 209.0040              | — .0213                  | 7.6488                  | — .0021     | 7.6467                   |
| 4              | .3150               | .6915      | 209.0065              | — .0238                  | .6235                   | — .0024     | .6211                    |
| 7              | .3151               | .6785      | 208.9936              | — .0109                  | .6366                   | — .0011     | .6355                    |
| 8              | .3047               | .6785      | 208.9832              | — .0005                  | .6262                   | — .0001     | .6261                    |
| 9              | .3063               | .6762      | 208.9825              | + .0002                  | .6301                   | + .0000     | .6301                    |
| 10             | .2992               | .6905      | 208.9897              | — .0070                  | .6087                   | — .0007     | .6080                    |
| 11             | .2783               | .6801      | 208.9584              | + .0243                  | .5982                   | + .0024     | .6006                    |
| 12             | .2743               | .6936      | 208.9679              | + .0148                  | .5807                   | + .0015     | .5822                    |
| 13             | .3015               | .7180      | 209.0195              | — .0368                  | .5835                   | — .0037     | .5798                    |
| 14             | .2956               | .7102      | 209.0058              | — .0231                  | .5854                   | — .0023     | .5831                    |
| 15             | .2905               | .6693      | 208.9598              | + .0229                  | .6212                   | + .0023     | .6235                    |
| 16             | .2950               | .6752      | 208.9702              | + .0125                  | .6198                   | + .0013     | .6211                    |
| 17             | .2910               | .6629      | 208.9539              | + .0288                  | .6281                   | + .0029     | .6310                    |
| 18             | .2972               | .7020      | 208.9992              | — .0165                  | .5952                   | — .0016     | .5936                    |
| 20             | .2885               | .6932      | 208.9817              | + .0010                  | .5953                   | + .0001     | .5954                    |
| 23             | .3087               | .6796      | 208.9883              | — .0056                  | .6291                   | — .0006     | .6285                    |
| 24             | .3024               | .6717      | 208.9741              | + .0086                  | .6307                   | + .0009     | .6316                    |
| 25             | .2968               | .6788      | 208.9756              | + .0071                  | .6180                   | + .0007     | .6187                    |
| 26             | .2991               | .6803      | 208.9794              | + .0033                  | .6188                   | + .0003     | .6191                    |
| 27             | .2885               | .6720      | 208.9605              | + .0222                  | .6165                   | + .0022     | .6187                    |
| Mean, 208.9827 |                     |            | Assumed Value, 7.6100 |                          |                         |             |                          |

TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS  $i$  AND  $j$ .

| Plate<br>No.   | Corrected Distance. |            | Sum<br>$i + j$ .      | Mean<br><i>minus</i> Sum | Difference<br>$i - j$ . | Scale Corr. | Corrected<br>Difference. |
|----------------|---------------------|------------|-----------------------|--------------------------|-------------------------|-------------|--------------------------|
|                | Star $i$ .          | Star $j$ . |                       |                          |                         |             |                          |
| 1              | 97.9511             | 93.3217    | 191.2728              | — .0390                  | 4.6294                  | — .0039     | 4.6255                   |
| 2              | .9621               | .3005      | .2626                 | — .0288                  | .6616                   | — .0029     | .6587                    |
| 4              | .9536               | .3102      | .2638                 | — .0300                  | .6434                   | — .0030     | .6404                    |
| 7              | .9062               | .3322      | .2384                 | — .0046                  | .5740                   | — .0005     | .5735                    |
| 8              | .9142               | .3137      | .2279                 | + .0059                  | .6005                   | + .0006     | .6011                    |
| 9              | .9135               | .3153      | .2288                 | + .0050                  | .5982                   | + .0005     | .5987                    |
| 10             | .9013               | .3124      | .2137                 | + .0201                  | .5889                   | + .0020     | .5909                    |
| 11             | .9102               | .3015      | .2117                 | + .0221                  | .6087                   | + .0022     | .6109                    |
| 12             | .9172               | .3025      | .2197                 | + .0141                  | .6147                   | + .0014     | .6161                    |
| 14             | .9327               | .3057      | .2384                 | — .0046                  | .6270                   | — .0005     | .6265                    |
| 15             | .9060               | .3139      | .2199                 | + .0139                  | .5921                   | + .0014     | .5935                    |
| 16             | .9038               | .3065      | .2103                 | + .0235                  | .5973                   | + .0024     | .5997                    |
| 18             | .9304               | .3179      | .2483                 | — .0145                  | .6125                   | — .0015     | .6110                    |
| 20             | .9121               | .3163      | .2284                 | + .0054                  | .5958                   | + .0005     | .5963                    |
| 21             | .9508               | .2985      | .2493                 | — .0155                  | .6523                   | — .0016     | .6507                    |
| 22             | .9449               | .3132      | .2581                 | — .0243                  | .6317                   | — .0024     | .6293                    |
| 23             | .9316               | .3025      | .2341                 | — .0003                  | .6291                   | — .0000     | .6291                    |
| 24             | .9298               | .2983      | .2281                 | + .0057                  | .6315                   | + .0006     | .6321                    |
| 25             | .9220               | .2925      | .2145                 | + .0193                  | .6295                   | + .0019     | .6314                    |
| 26             | .9248               | .2965      | .2213                 | + .0125                  | .6283                   | + .0012     | .6295                    |
| 27             | .9271               | .2926      | .2197                 | + .0141                  | .6345                   | + .0014     | .6359                    |
| Mean, 191.2338 |                     |            | Assumed Value, 4.6200 |                          |                         |             |                          |

TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS  $k$  AND  $l$ .

| Plate No.      | Corrected Distance. |            | Sum<br>$k + l$ | Mean<br><i>minus</i> Sum | Difference<br>$k - l$  | Scale Corr. | Corrected<br>Difference |
|----------------|---------------------|------------|----------------|--------------------------|------------------------|-------------|-------------------------|
|                | Star $k$ .          | Star $l$ . |                |                          |                        |             |                         |
| 1              | 76.4769             | 65.6116    | 142.0885       | — .0202                  | 10.8653                | — .0040     | 10.8613                 |
| 3              | .4789               | .6083      | .0872          | — .0189                  | .8706                  | — .0038     | .8668                   |
| 4              | .4862               | .6115      | .0977          | — .0294                  | .8747                  | — .0059     | .8688                   |
| 5              | .4861               | .6210      | .1071          | — .0388                  | .8651                  | — .0078     | .8573                   |
| 6              | .4750               | .6231      | .0981          | — .0298                  | .8519                  | — .0060     | .8459                   |
| 7              | .4821               | .5588      | .0409          | + .0274                  | .9233                  | + .0055     | .9288                   |
| 8              | .4786               | .5826      | .0612          | + .0071                  | .8960                  | + .0014     | .8974                   |
| 9              | .4756               | .5848      | .0604          | + .0079                  | .8908                  | + .0016     | .8924                   |
| 11             | .4723               | .5815      | .0538          | + .0145                  | .8908                  | + .0029     | .8937                   |
| 12             | .4578               | .5792      | .0370          | + .0313                  | .8786                  | + .0063     | .8849                   |
| 13             | .4914               | .6053      | .0967          | — .0284                  | .8861                  | — .0057     | .8804                   |
| 14             | .4915               | .5800      | .0715          | — .0032                  | .9115                  | — .0006     | .9109                   |
| 15             | .4720               | .5731      | .0451          | + .0232                  | .8989                  | + .0046     | .9035                   |
| 16             | .4787               | .5711      | .0498          | + .0185                  | .9076                  | + .0037     | .9113                   |
| 18             | .4886               | .5804      | .0690          | — .0007                  | .9082                  | — .0001     | .9081                   |
| 19             | .4958               | .5882      | .0840          | — .0157                  | .9076                  | — .0031     | .9045                   |
| 22             | .4828               | .5881      | .0709          | — .0026                  | .8947                  | — .0005     | .8942                   |
| 23             | .4679               | .6007      | .0686          | — .0003                  | .8672                  | — .0001     | .8671                   |
| 24             | .4654               | .6002      | .0656          | + .0027                  | .8652                  | + .0005     | .8657                   |
| 25             | .4576               | .5902      | .0478          | + .0205                  | .8674                  | + .0041     | .8715                   |
| 26             | .4693               | .5845      | .0538          | + .0145                  | .8848                  | + .0029     | .8877                   |
| 27             | .4630               | .5857      | .0487          | + .0196                  | .8773                  | + .0039     | .8812                   |
| Mean, 142.0583 |                     |            |                |                          | Assumed Value, 10.8860 |             |                         |



TABLE V.—PARALLAX EQUATIONS.

COMPARISON STARS *a* AND *b*.

| Plate. |               |                |                |       |     |       | <i>v</i> |
|--------|---------------|----------------|----------------|-------|-----|-------|----------|
| 1      | 1.00 <i>x</i> | —1.42 <i>y</i> | +1.49 <i>π</i> | —2.19 | = 0 | —2.61 |          |
| 4      | 1.00          | —1.41          | +1.56          | +0.09 | = 0 | —0.23 |          |
| 5      | 1.00          | —0.46          | +1.23          | —1.26 | = 0 | —1.11 |          |
| 6      | 1.00          | —0.46          | +1.23          | —2.68 | = 0 | —2.53 |          |
| 7      | 1.00          | —0.04          | —0.38          | +5.40 | = 0 | +3.93 |          |
| 8      | 1.00          | +0.01          | —0.89          | +1.48 | = 0 | —0.58 |          |
| 10     | 1.00          | +0.01          | —0.96          | +2.78 | = 0 | +0.64 |          |
| 11     | 1.00          | +0.50          | +0.85          | —2.66 | = 0 | —2.10 |          |
| 12     | 1.00          | +0.50          | +0.85          | —1.00 | = 0 | —0.44 |          |
| 13     | 1.00          | +0.55          | +1.29          | +0.01 | = 0 | +1.17 |          |
| 14     | 1.00          | +0.55          | +1.29          | +2.27 | = 0 | +3.43 |          |
| 15     | 1.00          | +1.02          | —1.08          | +3.98 | = 0 | +2.63 |          |
| 16     | 1.00          | +1.03          | —1.11          | +3.07 | = 0 | +1.69 |          |
| 17     | 1.00          | +1.03          | —1.11          | +2.52 | = 0 | +1.14 |          |
| 18     | 1.00          | +1.54          | +1.20          | —0.49 | = 0 | +1.48 |          |
| 19     | 1.00          | +1.54          | +1.20          | +0.39 | = 0 | +2.36 |          |
| 20     | 1.00          | +1.55          | +1.31          | —1.55 | = 0 | +0.56 |          |
| 21     | 1.00          | +1.55          | +1.31          | —1.02 | = 0 | +1.09 |          |
| 22     | 1.00          | +1.56          | +1.33          | —1.58 | = 0 | +0.57 |          |
| 23     | 1.00          | +1.96          | —0.39          | —2.30 | = 0 | —1.92 |          |
| 24     | 1.00          | +1.96          | —0.39          | —3.67 | = 0 | —3.29 |          |
| 25     | 1.00          | +1.97          | —0.48          | —1.64 | = 0 | —1.37 |          |
| 26     | 1.00          | +1.97          | —0.48          | —2.86 | = 0 | —2.59 |          |
| 27     | 1.00          | +1.97          | —0.57          | —2.05 | = 0 | —1.89 |          |

$$\Sigma v^2 = 95.45.$$

## Normal Equations.

$$\begin{aligned} +24.0000x + 18.9800y + 8.3000\pi - 4.9600 &= 0 \\ +40.0044 - 1.2806 - 17.3438 &= 0 \\ +27.1124 - 24.6113 &= 0 \end{aligned}$$

## Solution.

| In units of 2d dec. place. | In Arc.                        |
|----------------------------|--------------------------------|
| $\pi = +1.2446 \pm 0.3080$ | $\pi = +0.3486'' \pm 0.0863''$ |
| $y = +0.9276 \pm 0.2877$   | $y = +0.2598 \pm 0.0806$       |
| $x = -0.9574 \pm 0.2936$   | $x = -0.2682 \pm 0.0822$       |

Scale.                  Arc.

$$\text{Probable error of one equation} = \pm 1.438 = \pm 0''.403.$$

TABLE V.—PARALLAX EQUATIONS.

COMPARISON STARS  $c$  AND  $d$ .

| Plate. |          |           |             |       |     | $p$   |
|--------|----------|-----------|-------------|-------|-----|-------|
| 4      | 1.00 $x$ | -1.41 $y$ | +1.76 $\pi$ | -1.68 | = 0 | -0.67 |
| 6      | 1.00     | -0.46     | +1.48       | -4.13 | = 0 | -2.94 |
| 7      | 1.00     | -0.04     | -0.65       | +3.05 | = 0 | +1.56 |
| 8      | 1.00     | +0.01     | -1.15       | +1.83 | = 0 | -0.32 |
| 9      | 1.00     | +0.01     | -1.15       | +2.25 | = 0 | +0.10 |
| 10     | 1.00     | +0.01     | -1.23       | +2.73 | = 0 | +0.47 |
| 11     | 1.00     | +0.50     | +1.12       | -2.06 | = 0 | -0.80 |
| 12     | 1.00     | +0.50     | +1.12       | -1.88 | = 0 | -0.62 |
| 15     | 1.00     | +1.02     | -1.34       | +3.12 | = 0 | +1.31 |
| 16     | 1.00     | +1.03     | -1.36       | +3.84 | = 0 | +2.01 |
| 17     | 1.00     | +1.03     | -1.36       | +4.05 | = 0 | +2.22 |
| 18     | 1.00     | +1.54     | +1.46       | +0.08 | = 0 | +2.42 |
| 19     | 1.00     | +1.54     | +1.46       | -1.39 | = 0 | +0.95 |
| 20     | 1.00     | +1.55     | +1.55       | +0.80 | = 0 | +3.27 |
| 22     | 1.00     | +1.56     | +1.57       | -2.81 | = 0 | -0.31 |
| 23     | 1.00     | +1.96     | -0.67       | +0.50 | = 0 | +0.16 |
| 24     | 1.00     | +1.96     | -0.67       | -2.73 | = 0 | -3.07 |
| 25     | 1.00     | +1.97     | -0.76       | -2.15 | = 0 | +2.61 |
| 26     | 1.00     | +1.97     | -0.76       | -0.66 | = 0 | -1.02 |
| 27     | 1.00     | +1.97     | -0.84       | -1.44 | = 0 | -2.01 |

$$\Sigma v^2 = 62.46$$

Normal Equations.

$$\begin{aligned} +20.0000x + 18.2200y - 0.4200\pi + 1.3200 &= 0 \\ +34.7690 - 4.1472 - 4.3505 &= 0 \\ +29.7612 - 38.6706 &= 0 \end{aligned}$$

Solution.

| In units of 2d dec. place. | In Arc.                        |
|----------------------------|--------------------------------|
| $\pi = +1.3735 \pm 0.3002$ | $\pi = +0.''3847 \pm 0.''0841$ |
| $y = +0.5902 \pm 0.3036$   | $y = +0.1653 \pm 0.0850$       |
| $x = -0.5748 \pm 0.2828$   | $x = -0.1610 \pm 0.0792$       |

|                                              |                   |
|----------------------------------------------|-------------------|
| Scale.                                       | Arc.              |
| Probable error of one equation = $\pm 1.293$ | = $\pm 0''.362$ . |

TABLE V.—PARALLAX EQUATIONS.

COMPARISON STARS *c* AND *f*.

| Plate. |      |                |         |        |     |        |  |
|--------|------|----------------|---------|--------|-----|--------|--|
| 1      | 1.00 | —1.42 <i>y</i> | + 1.047 | + 1.39 | = 0 | + 0.53 |  |
| 6      | 1.00 | —0.46          | + 1.30  | + 0.77 | = 0 | + 1.74 |  |
| 8      | 1.00 | + 0.01         | —1.44   | + 4.23 | = 0 | + 0.27 |  |
| 9      | 1.00 | + 0.01         | —1.44   | + 4.87 | = 0 | + 0.28 |  |
| 11     | 1.00 | + 0.50         | + 1.52  | —2.86  | = 0 | —0.15  |  |
| 12     | 1.00 | + 0.50         | + 1.52  | —3.64  | = 0 | —0.93  |  |
| 13     | 1.00 | + 0.55         | + 1.25  | —6.24  | = 0 | —4.01  |  |
| 14     | 1.00 | + 0.55         | + 1.25  | —3.40  | = 0 | —1.17  |  |
| 15     | 1.00 | + 1.02         | —1.33   | + 3.32 | = 0 | + 0.95 |  |
| 16     | 1.00 | + 1.03         | —1.32   | + 1.31 | = 0 | —1.02  |  |
| 17     | 1.00 | + 1.03         | —1.32   | + 1.21 | = 0 | —1.12  |  |
| 18     | 1.00 | + 1.54         | + 1.32  | —4.19  | = 0 | —0.48  |  |
| 20     | 1.00 | + 1.55         | + 1.23  | —1.73  | = 0 | + 1.81 |  |
| 22     | 1.00 | + 1.56         | + 1.21  | —0.58  | = 0 | + 2.93 |  |
| 23     | 1.00 | + 1.96         | —1.58   | + 1.37 | = 0 | —0.24  |  |
| 24     | 1.00 | + 1.96         | —1.58   | + 2.31 | = 0 | + 0.70 |  |
| 25     | 1.00 | + 1.97         | —1.56   | + 0.42 | = 0 | —1.14  |  |
| 27     | 1.00 | + 1.97         | —1.55   | + 1.99 | = 0 | + 0.45 |  |

$$\Sigma r^2 = 39.13$$

## Normal Equations.

$$\begin{aligned} +18.0000r + 15.8300y - 1.4800\pi + 0.5500 &= 0 \\ +29.1481 &- 9.7778 &- 2.8854 &= 0 \\ +34.4666 &- 58.2440 &= 0 \end{aligned}$$

## Solution.

| In units of 2d dec. place. | In Arc.                  |
|----------------------------|--------------------------|
| $\pi = +2.0277 + 0.2001$   | $\pi = +0.5680 + 0.0560$ |
| $y = +1.3500 + 0.2792$     | $y = +0.3781 + 0.0782$   |
| $x = -1.0511 + 0.2567$     | $x = -0.2044 + 0.0719$   |

|                                | Scale.   | Arc.       |
|--------------------------------|----------|------------|
| Probable error of one equation | = +1.089 | = +0.00305 |

TABLE V.—PARALLAX EQUATIONS.

| COMPARISON STARS $g$ AND $h$ . |          |           |             |       |     |                    |
|--------------------------------|----------|-----------|-------------|-------|-----|--------------------|
| Plate.                         |          |           |             |       |     | $\rho$             |
| 1                              | 1.00 $x$ | —1.42 $y$ | +0.17 $\pi$ | +3.67 | = 0 | +2.75              |
| 4                              | 1.00     | —1.41     | +0.04       | +1.11 | = 0 | —0.11              |
| 7                              | 1.00     | —0.04     | —1.01       | +2.55 | = 0 | —0.02              |
| 8                              | 1.00     | +0.01     | —0.76       | +1.61 | = 0 | —0.29              |
| 9                              | 1.00     | +0.01     | —0.76       | +2.01 | = 0 | +0.11              |
| 10                             | 1.00     | +0.01     | —0.70       | —0.20 | = 0 | —1.96              |
| 11                             | 1.00     | +0.50     | +0.84       | —0.94 | = 0 | +1.31              |
| 12                             | 1.00     | +0.50     | +0.84       | —2.78 | = 0 | —0.53              |
| 13                             | 1.00     | +0.55     | +0.42       | —3.02 | = 0 | —1.72              |
| 14                             | 1.00     | +0.55     | +0.42       | —2.69 | = 0 | —1.39              |
| 15                             | 1.00     | +1.02     | —0.59       | +1.35 | = 0 | +0.65              |
| 16                             | 1.00     | +1.03     | —0.56       | +1.11 | = 0 | +0.49              |
| 17                             | 1.00     | +1.03     | —0.56       | +2.10 | = 0 | +1.48              |
| 18                             | 1.00     | +1.54     | +0.52       | —1.64 | = 0 | +0.72              |
| 20                             | 1.00     | +1.55     | +0.40       | —1.46 | = 0 | +0.62              |
| 23                             | 1.00     | +1.96     | —1.10       | +1.85 | = 0 | +0.73              |
| 24                             | 1.00     | +1.96     | —1.10       | +2.16 | = 0 | +1.04              |
| 25                             | 1.00     | +1.97     | —1.05       | +0.87 | = 0 | —0.13              |
| 26                             | 1.00     | +1.97     | —1.05       | +0.91 | = 0 | —0.09              |
| 27                             | 1.00     | +1.97     | —0.99       | +0.87 | = 0 | +0.02              |
|                                |          |           |             |       |     | $\Sigma v^2=23.80$ |

Normal Equations.

$$\begin{aligned}
 +20.0000x + 15.2600y - 6.5800\pi + 9.4400 &= 0 \\
 +32.3736 - 9.7115 + 1.1300 &= 0 \\
 +11.4706 - 21.2136 &= 0
 \end{aligned}$$

Solution.

| In units of 2d dec. place.  | In Arc.                     |
|-----------------------------|-----------------------------|
| $\pi = + 2.3631 \pm 0.2780$ | $\pi = + 0.6619 \pm 0.0779$ |
| $y = + 0.8277 \pm 0.1753$   | $y = + 0.2318 \pm 0.0491$   |
| $x = - 0.1459 \pm 0.1760$   | $x = - 0.4087 \pm 0.0493$   |

Scale. Arc.

Probable error of one equation =  $\pm 0.798 = \pm 0''.274$

TABLE V.—PARALLAX EQUATIONS.

| COMPARISON STARS $i$ AND $j$ . |      |          |            |         |       |                      |
|--------------------------------|------|----------|------------|---------|-------|----------------------|
| Plate.                         |      |          |            |         |       | $v$                  |
| 1                              | 1.00 | $-1.42y$ | $-1.05\pi$ | $+0.55$ | $= 0$ | $-0.23$              |
| 2                              | 1.00 | $-1.42$  | $-1.05$    | $+3.87$ | $= 0$ | $+3.09$              |
| 4                              | 1.00 | $-1.41$  | $-1.15$    | $+2.04$ | $= 0$ | $+1.02$              |
| 7                              | 1.00 | $-0.04$  | $-0.12$    | $-4.65$ | $= 0$ | $-3.79$              |
| 8                              | 1.00 | $+0.01$  | $+0.38$    | $-1.89$ | $= 0$ | $-0.13$              |
| 9                              | 1.00 | $+0.01$  | $+0.38$    | $-2.13$ | $= 0$ | $-0.11$              |
| 10                             | 1.00 | $+0.01$  | $+0.47$    | $-2.91$ | $= 0$ | $-0.68$              |
| 11                             | 1.00 | $+0.50$  | $-0.32$    | $-0.91$ | $= 0$ | $-0.73$              |
| 12                             | 1.00 | $+0.50$  | $-0.32$    | $-0.39$ | $= 0$ | $-0.21$              |
| 14                             | 1.00 | $+0.55$  | $-0.81$    | $+0.65$ | $= 0$ | $-0.35$              |
| 15                             | 1.00 | $+1.02$  | $+0.60$    | $-2.65$ | $= 0$ | $-0.51$              |
| 16                             | 1.00 | $+1.03$  | $+0.62$    | $-2.03$ | $= 0$ | $+0.15$              |
| 18                             | 1.00 | $+1.54$  | $-0.71$    | $-0.90$ | $= 0$ | $-2.06$              |
| 20                             | 1.00 | $+1.55$  | $-0.84$    | $-2.37$ | $= 0$ | $-3.83$              |
| 21                             | 1.00 | $+1.55$  | $-0.84$    | $+3.07$ | $= 0$ | $+1.60$              |
| 22                             | 1.00 | $+1.56$  | $-0.86$    | $+0.93$ | $= 0$ | $-0.59$              |
| 23                             | 1.00 | $+1.96$  | $-0.11$    | $+0.91$ | $= 0$ | $+1.00$              |
| 24                             | 1.00 | $+1.96$  | $-0.11$    | $+1.21$ | $= 0$ | $+1.30$              |
| 25                             | 1.00 | $+1.97$  | $-0.02$    | $+1.14$ | $= 0$ | $+1.44$              |
| 26                             | 1.00 | $+1.97$  | $-0.02$    | $+0.95$ | $= 0$ | $+1.25$              |
| 27                             | 1.00 | $+1.97$  | $+0.06$    | $+1.59$ | $= 0$ | $+2.08$              |
|                                |      |          |            |         |       | $\Sigma v^2 = 58.95$ |

## Normal Equations.

$$\begin{aligned}
 +21.0000x + 15.3700y - 5.8200\pi - 3.9200 &= 0 \\
 +37.8627x - 0.3251\pi - 1.5679 &= 0 \\
 +8.3404x - 13.2118 &= 0
 \end{aligned}$$

## Solution.

In units 2d dec. place.

In Arc.

$$\begin{aligned}
 \pi &= +2.3578 \pm 0.4923 & \pi &= +0.6604 \pm 0.1379 \\
 y &= -0.3975 \pm 0.2366 & y &= -0.1113 \pm 0.0663 \\
 x &= +1.1310 \pm 0.2664 & x &= +0.3168 \pm 0.0746
 \end{aligned}$$

Scale.

Arc.

$$\text{Probable error of one equation} = \pm 1.221 = \pm 0''.342$$



TABLE V.—PARALLAX EQUATIONS.

| COMPARISON STARS $k$ AND $l$ . |          |           |             |       |     | $v$   |
|--------------------------------|----------|-----------|-------------|-------|-----|-------|
| Plate.                         |          |           |             |       |     |       |
| 1                              | 1.00 $x$ | -1.42 $y$ | +1.11 $\pi$ | -2.47 | = 0 | -1.13 |
| 3                              | 1.00     | -1.41     | +1.21       | -1.92 | = 0 | -0.47 |
| 4                              | 1.00     | -1.41     | +1.21       | -1.72 | = 0 | -0.27 |
| 5                              | 1.00     | -0.46     | +0.81       | -2.87 | = 0 | -2.08 |
| 6                              | 1.00     | -0.46     | +0.81       | -4.01 | = 0 | -3.22 |
| 7                              | 1.00     | -0.04     | +0.06       | +4.28 | = 0 | +4.17 |
| 8                              | 1.00     | +0.01     | -0.45       | +1.14 | = 0 | +0.47 |
| 9                              | 1.00     | +0.01     | -0.45       | +0.64 | = 0 | -0.03 |
| 11                             | 1.00     | +0.50     | +0.39       | +0.77 | = 0 | +0.87 |
| 12                             | 1.00     | +0.50     | +0.39       | -0.11 | = 0 | -0.01 |
| 13                             | 1.00     | +0.55     | +0.88       | -0.56 | = 0 | +0.05 |
| 14                             | 1.00     | +0.55     | +0.88       | +2.49 | = 0 | +3.10 |
| 15                             | 1.00     | +1.02     | -0.66       | +1.75 | = 0 | +0.61 |
| 16                             | 1.00     | +1.03     | -0.68       | +2.53 | = 0 | +1.36 |
| 18                             | 1.00     | +1.54     | +0.77       | +2.21 | = 0 | +2.45 |
| 19                             | 1.00     | +1.54     | +0.77       | +1.85 | = 0 | +2.09 |
| 22                             | 1.00     | +1.56     | +0.92       | +0.82 | = 0 | +1.22 |
| 23                             | 1.00     | +1.96     | +0.05       | -1.89 | = 0 | -2.51 |
| 24                             | 1.00     | +1.96     | +0.05       | -2.03 | = 0 | -2.65 |
| 25                             | 1.00     | +1.97     | -0.04       | -1.45 | = 0 | -2.17 |
| 26                             | 1.00     | +1.97     | -0.04       | +0.17 | = 0 | -0.55 |
| 27                             | 1.00     | +1.97     | -0.12       | -0.48 | = 0 | -1.29 |

$$\Sigma v^2 = 78.32$$

Normal Equations.

$$\begin{aligned} +22.0000x + 13.4400y + 7.8700\pi - 0.8600 &= 0 \\ +36.1266 - 2.1518 + 13.8148 &= 0 \\ +10.6869 - 10.3893 &= 0 \end{aligned}$$

Solution.

| In units of 2d dec. place. | In Arc.                    |
|----------------------------|----------------------------|
| $\pi = +1.0603 \pm 0.5529$ | $\pi = +0.2970 \pm 0.1549$ |
| $y = -0.2493 \pm 0.2591$   | $y = -0.0698 \pm 0.0726$   |
| $x = -0.1879 \pm 0.2919$   | $x = -0.0526 \pm 0.0818$   |

Scale. Arc.  
Probable error of one equation =  $\pm 1.370 = \pm 0.1384$



"There shall be appended to each dissertation a statement of the educational institutions that the author has attended, a list of the degrees and honors conferred upon him, as well as the titles of any previous publications." *Extract from Section VI.—Regulations for the Degree of Doctor of Philosophy, established by the University Council.*

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